Deposits and Loans Interest Rates Lag Structure and Business Cycles (Case Study of United States)

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Abstract

In addition to discuss economic cycle theories in this paper, by decomposing money market into two markets of "saving-depositing" and "investment-credit", we study the time structure behavior of depositor-bank-investor and conclude that the conventional banking structure creates fluctuations in money sector and interest rates. These fluctuations affect real sector through saving and investment and fluctuate the economy. Mathematical derivations show that conventional banking structure is a source of economic cycles and fluctuations.

To show the oscillatory natures of the interest rates we tested second order linear difference structure of time paths for 10 different short, medium and long terms interest rates of Unites States of America for long samples. We observed that the short term interest rates are the source of oscillation and oscillation will be transferred to real economy. We conclude that the source of oscillation is emanated from interest rates to real sector, and the interest rates are sources of business cycles in the economy.

Empirical investigations shows all short term interest rates' second order homogeneous linear difference equations have complex characteristic roots; but the roots for medium and long terms interest rates are real. This proves that the source of fluctuations in real economy comes from short term interest rates, though medium and long terms interest rates have real characteristic roots and thus dampening effects on real economy.

Keywords: *Usury-free banking, Lag structure, Business cycle, Interest rate.*

Introduction

Countries of the world have always experienced business cycles in the economy with a sinusoidal trend. These fluctuations which are the result of the capitalistic nature of the economy, spreads over other countries through international transactions of goods and capital. Practically, the price signals of commodities and various capital yields create the global flow and the movement of business cycles. Various business cycles caused by weekly, monthly or seasonal and cycles caused by inventory fluctuations which last less than half a decade and are called "Kitchin"[2] cycles are not considered in this paper. Fluctuations considered here are economic or business cycles which happen in a decade and turns the economy from prosperity to recession and crisis, and again, to prosperity usually within a decade. The

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[2] -Kitchin, Joseph (1923). "Cycles and Trends in Economic Factors". Review of Economics and Statistics 5 (1). 10–16. http://www.jstor.org/stable/1927031

large cycles which are usually caused by structural changes and occur within two decades, are reputed as "Kuznets"[3] cycles, and longer fluctuations caused by science and technology developments and take about half a century to complete, are called "Kondratieff"[4] cycles. Sometimes, the overlap of these cycles with usual business cycles creates large global crises such as crisis of 1920 decade and the crisis of the end of 2000 decade.

This waviness is the result of the nature of mankind behavior; that is to say, the natural manner of men causes this behavior. This is because human being is extravagant and when he is rich, shows more extravagance, but the limited available sources do not let him to continue. Therefore, it has to stop during the economic prosperity. That is because resources do not increase proportional with people's economic activities. The scarcity of resources causes price increase and decrease of profit margin which is the beginning of recession and the turning point of business cycle.

Business cycle theories

For the first time, Juglar[5] realized this phenomenon in 1862 for 8-11 years periods. After that, several theories were introduced about business cycles which studied business cycle from different points of view. As Schumpeter[6] described the four stages of business cycle, during prosperity stage, increase in production and prices and the decrease of interest rate are seen. During the second stage or recession, production and prices will decrease but interest rate will start to increase until we reach to the third stage which is the crisis when stock markets collapse and firms become bankrupt. Recovery starts during the fourth stage and is accompanied by stock exchange prosperity, increase of output, demand and prices.

"Goodwin"[7] believes that the reason for business cycles is the gap between income distribution between the profit of investors of economic firms and the earnings of labor force; when the economy has a high employment rate and labor demand increases, workers can't ask for higher wages because labor contracts are annual or have a fixed time and wages can be changed after the end of the contract period. The reverse happens during recession. Therefore, the income of the labor force is adjusted with the income of capital factor after a time lag which creates a cyclical behavior for matching production with consumption and ultimately shapes the cycle. Although Goodwin's theory has time dynamism, but mathematically, it's simple form gives first order difference equations. Although these equations have time ascending or descending trend, and are converging or diverging trends, but cannot obtain time oscillatory trends. The difference equation should be of second order to create sinusoid trends.

Some economists believe that the reason for business cycles is technological shocks[8], some other believe that they are created by political party and political decisions cycles [9]. Marxists believe it as the essence of capitalism, and New Classics believe that the decrease of labor purchasing power is the reason for capitalistic crises.

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[3]- Simon Kuznets, Secular Movements in Production and Prices. Their Nature and their Bearing upon Cyclical Fluctuations. Boston: Houghton Mifflin, 1930.
[4] - Kondratieff, N. D.; Stolper, W. F. (1935). "The Long Waves in Economic Life". Review of Economics and Statistics 17 (6): 105–115. http://www.jstor.org/stable/1928486
[5] -Clement Juglar, Des Crises commerciales et leur retour periodique en France, en Angleterre, et aux Etats-Unis. Paris: Guillaumin, 1862. http://gallica.bnf.fr/ark:/12148/bpt6k1060720
[6] - Schumpeter, J. A. (1954). History of Economic Analysis. London: George Allen & Unwin.
[7]- Richard Goodwin, "The Business Cycle as a Self-Sustaining Oscillation", 1949, Econometrica.
Richard Goodwin, Nonlinear Dynamics and Economic Evolution", 1991, in Niels Thygesen et al., editors, Business Cycles.
[8] - Real business cycle theory. Kydland, Finn E.; Prescott, Edward C. (1982). "Time to Build and Aggregate Fluctuations". Econometrica 50 (6): 1345–1370. http://www.jstor.org/stable/1913386
[9] -Political business cycle. Partisan business cycle. Michal Kalecki.

By scrutinizing the nature of waving and sinusoid movement of economic activities we can realize this nature automatically leads to situations in which economy turns from prosperity to recession and crisis and again into prosperity. Some believe the reason behind this sinusoid movement is inventory operation and supply flow. That is, when the economy produces more than consumption, goods are accumulated in the warehouses and the producer has to decrease price to sell the inventory. The lowering price and the large accumulated inventory will loosen production and producer has to decrease production which decreases the income of factors of production (labor, capital and others). That is to say, income at macro level will drop which decreases demand for goods and services. The lower demand will decrease price and economic firms face more recession. This phenomenon goes forward until recession changes to crisis. During this stage, production continues to decrease, but practically the trend of price decrease is lowering or stopped. Price reaches production cost and since it is not possible for the producer to decrease price below production cost, production is stopped and many inefficient firms become bankrupt and total supply will decline in the country. The decrease in supply is followed by price increase and persuades production which increases income of factors of production in the next stage and is followed by more price increase and moves the economy from crisis to prosperity until we reach the beginning of next business cycle. After a while, recession starts and the cycle reiterate again.

According to Keynesian Economists, fluctuation in total demand causes the economy to reach equilibrium in short period which is not equilibrium at full employment. The motivation to obtain full employment in equilibrium which is below full employment and the inefficient excessive use of sources and factors of production and production capacity in equilibriums beyond full employment will cause business cycles. Keynesian theories believe that the lack of enough effective demand in the economy is an indigenous cause for crises while Classic and New-Classics believe exogenous factors are the causes of business cycles. They believe supply will create its own demand (Say's Law). Since according to these two views, government interference policies have positive or negative effect in abolishing crises, will lead to different policy results. The first offers financial and the second offers monetary policy. Paul Samuelson's Oscillator Model describes Keynesian analysis on the basis of multiplier effect (on consumption) and accelerator (in investment) which create cycles through changes of total demand components. The struggle between Keynesian and Classic economists can be introduced in this discussion. Keynes believed when the economy is in liquidity trap, or there is no coordination between investment and saving, in order to obtain equilibrium in money market, it is necessary to reduce interest rate below zero. In this situation it is not possible to use monetary policies to overcome crisis because, interest rate is so low that cannot be reduced. Essentially, the liquidity trap is because of poor relationship between efficiency rate of production in real sector and interest rate. That is to say, the interest rate has decreased to a very low figure, regardless of productivity rate. One of the reasons behind this situation is the increase of investment risk which decreases the net investment yield. The point is that, if it was possible to make negative interest rate in the economy, which is not possible operationally, monetary policies were capable of overcoming crisis. Therefore, national policies are recommended which are conducted in group 20 by injecting about $5 trillion to overcome the present crisis. This method practically causes the crisis stricken economies come out the crisis, but in order to achieve this effect it is necessary to wait about half a decade until economic mechanisms lead the economy to prosperity through natural ways. The overview of economic variables in the last decade shows the same in comparison with 1920 crisis. The sever oil and grains price increase and successive draughts and unexpected ecological and meteorological events had increasing effects on this crisis. Similar conditions of the present crisis were also seen in the crisis of 1920 decade. Also during that period interest rate was severely reduced, but the economic conditions were so bad that even the low price of capital could not allocate adequate resources for production. Therefore, similar to financial policy which took about a decade to recover the global economy from crisis, accordingly at present time we have to wait until the injection of financial resources could again turn the global economy to recovery and prosperity.

Money market and the role of banks in creation of economic cycles

The essential solution which is forgotten by economists is the reform of monetary and banking structure. Some theories suggest that banking structures are among factors which create crisis. In spite of excessive inventory theory, the variation of inventory is the effect of cycles, not its cause. In contrary to what is formed in the minds of economists around the world, the recent recession was not because of recession in American housing market; rather it was the consequence of this crisis. In spite of the set fort theories, the cause of recession was not in housing sector, but it was the result of a structural behavior in money and banking sector. The theory of credit cycles by Irving Fisher[10] is one of the interesting theories about the cause of business cycles. He believes that credit cycles are the starting reason for economic cycles. Accordingly, the net increase of credits and equal with that, the debts as a percentage of GDP, creates economic prosperity, and vice versa, the decrease of net credit, moves the economy towards recession and crisis. In direction of Fisher theory, Minsky[11] puts forward the financial instability theory and develops Fisher theory by describing credit bubbles and the burst of these bubbles and their effects on economic cycles. He believes that the reason is the accumulated debts to banks. In this connection the Austrian school of thought[12] can be put forward which believes that the cause of credit changes is the monetary expansionary policies of central banks. This school refers to the role of interest rate as the price of capital for investment and agrees that in an open economy without central bank, interest rate describes the real time preference of borrowers and lenders, but central bank disturbs this equilibrium between them and inevitably creates fluctuations in the economy. When Fisher put forward this theory, the dynamic mathematical tools such as difference equations were not used. If the application of difference equations which was set forth in 1950 decade were introduced into economic analysis a few decades earlier, by using this tool, Fisher could dynamize the occurrence of crisis, as he explained the richest monetary theory which is the quantitative theory of money with the help of Balance Law from physics[13].

In spite of the view of recent economists who believe in a single money market, we divide it into two separate markets where banks operate as intermediates between them. In other words, the demand of the bank for deposits is at one side of the bank which intersects with the supply of deposits and fixes interest rate which is called deposit interest rate. On the other side, bank creates another market by supplying funds for credit finance and its intersection with demand for credit facilities creates the interest rate for credit facilities. That is to say bank stands between two markets of supply and demand of monetary funds. Now suppose that because of the consumption increase, deposit supply falls down. This will increase the deposit interest rate. The increase in deposit interest rate cannot instantly increase credit interest rate because credit contracts have been fixed for a period of time and bank has to wait for the duration of the contract before increasing the rate in new contracts for credit facilities and subsequently, the interest rate for credit facilities will increase in the economy. Bank will face losses during this period of time and after a time lag will compensate it by the increase of credit facility rate. Although this lag is not quite visible for people, from economic point of view it creates a special dynamic relationship between supply and demand for capital. It can mathematically be shown that because of this lag, the relationship between these two variables is a second degree behavioral difference equation. Second degree difference equations have a wavy character which creates cycles. Therefore, practically, the cycles created in the saving and families' consumption will transfer into the investment and production sectors.

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[10] - Fisher, Irving (1933), "The Debt-Deflation Theory of Great Depressions", Econometrica, http://fraser.stlouisfed.org/docs/meltzer/fisdeb33.pdf

[11] Hyman Minsky The Credit Crisis: Denial, delusion and the "defunct" American economist who foresaw the dénouement. http://www.finfacts.ie/irishfinancenews/article_1014734.shtml

Hyman Minsky The Financial Instability Hypothesis, Working Paper No 74, May 1992, pp. 6-8. http://www.levy.org/pubs/wp74.pdf

[12]- Works of Ludwig von Mises and Friedrich Hayek.

[13] - William Jack Baumol, Topology of Second Order Linear Difference Equations with Constant Coefficients, 1958, Econometrica

William Jack Baumol, Economic Dynamics, R. Turvey, 1951.

In other words, all fluctuations seen in real sector of the economy are induced by fluctuations in the money market. Many studies have been carried out about this subject which is one of the obvious subjects of monetary theories in the economy. If fluctuations of the money sector of the economy alleviate, many of the fluctuations of the real sector will also tend to be stabilized. The most important effect of elimination of "usury" is the direct bridging of the investment sector to saving sector of the economy[14]. When banks try to maximize profit through optimizing their behavior, the intermediate sector (banks) acts as a separate sector of the economy and the differences created by them between interest rates of supply and demand of funds, create fluctuations in financial markets. Since loan contracts have maturity periods, changes in supply (sources) or demand (uses) side of funds take time to be transferred to the other side. This time lag creates continuous fluctuations in financial markets. This analysis can be seen in the following graph:

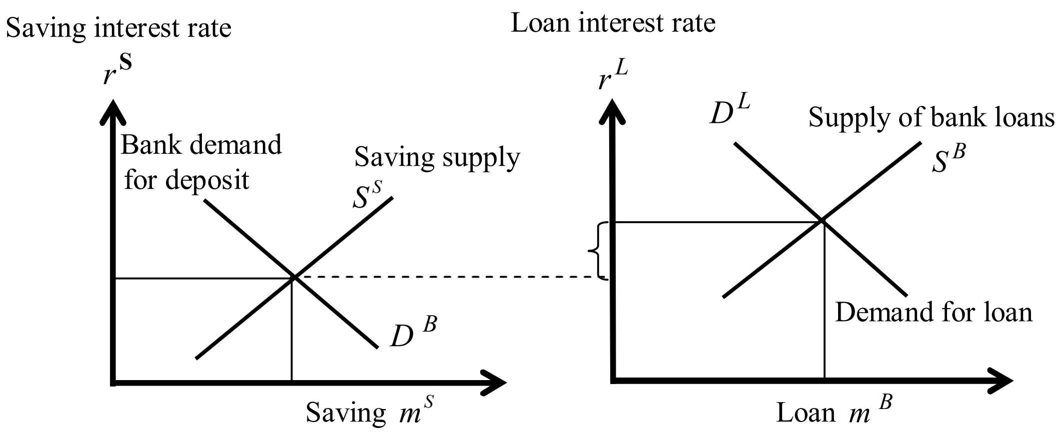


Figure 1

Where:

S^B	Loan supply by banks
S^S	Fund supply by depositors
D^L	Loan fund demand
D^B	Bank demand for funds
r^S	Saving interest rate
r^L	Loan interest rate
m^S	Amount of saving
m^B	Amount of loans
R	Bank's revenue

Bank's revenue at time t is equal to:

$$R_t = m_t^B r_t^L - m_t^S r_t^S \tag{1}$$

[14] - For more information look through: Bidabad, Bijan, Economic-juristic analysis of usury in consumption and investment loans and contemporary jurisprudence shortages in exploring legislator commandments. Proceeding of the 2nd International Islamic Banking Conference. Monash University of Malaysia. 9-10 September 2004. Reprinted in: National Interest, Journal of the Center for Strategic Research, Vol. 2, No. 1, winter 2006, pp. 72-90. Tehran, Iran. http://www.bidabad.com/doc/reba-en.pdf

At equilibrium we have:

$$m_t^B = D_t^L = S_t^B \quad (2)$$

$$m_t^S = D_t^B = S_t^S \quad (3)$$

Now suppose a new condition in which demand for loans decreases and D_t^L moves to left side to D_{t+1}^L. In the new equilibrium, if bank's revenue turns negative, we have:

$$r_{t+1}^L < r_t^L \quad (4)$$

Or:

$$R_{t+1} = m_{t+1}^B r_{t+1}^L - m_t^S r_t^S < 0 \quad (5)$$

Therefore, regarding time-loan contracts of bank, bank has practically to compensate losses during t+1 period from other sources until the next period when D^B curve moves to the left hand side. That is to say:

$$r_{t+2}^S > r_{t+1}^L \quad (6)$$

$$R_{t+2} = m_{t+1}^B \ r_{t+1}^L - m_{t+2}^S \ r_{t+2}^S > 0 \quad (7)$$

By generalizing this hypothesis we clearly see that whenever shocks occur in deposit supply or demand for banks' credit facilities (loan), because of time contracts, these shocks will be transferred to the other market in the next period and the fluctuations transfer from market to market alternatively and permanently fluctuates other related markets.

By considering the sign of three equations of (1), (5) and (7), we can clearly see that the behavior of variable R is alternative in different time periods. The behavior of the two markets described above can be drowned according to Cob-Web model which creates different fluctuation according to the gradient of different parts of supply and demand curves.

The interest rates in the two markets are as follows:

$$r_t^S = r^S(m_t^S) \quad (8)$$

$$r_t^L = r^L(m_t^B) \quad (9)$$

If according to the above assumptions we adjust the relationship of the two markets with one time-lag, we have:

$$m_{t+1}^S = f(m_t^B) \quad (10)$$

By replacing (8) and (9) in (10), we have:

$$r_t^S = r^S(f(m_{t-1}^B)) = r^S(f(r^{L^{-1}}(r_{t-1}^L))) \quad (11)$$

In other words, the interest rate in the deposit market is a function of interest rate in loan market in the last period. The adjustment takes place when the return movement occurs in the next period which means that the interest rate of loan market is itself a function of interest rate of deposit market in the previous period, or:

$$m_{t+1}^B = g(m_t^S) \quad (12)$$

By replacing (10) in (12), we will have:

$$m_{t+1}^{B} = g(f(m_{t-1}^{B})) \quad (13)$$

This is a second order difference equation which is characterized to fluctuate easily in time. This is also true about interest rates. By replacing (12) in (10), we have:

$$m_{t+1}^{S} = f(g(m_{t-1}^{S})) \quad (14)$$

This equation similar to equation (13) can be completely oscillatory. By replacing (12) in (9), we will have:

$$r_t^{L} = r^{L}(g(m_{t-1}^{S})) = r^{L}(g(r^{S^{-1}}(r_{t-1}^{L}))) \quad (15)$$

Since equations (15) and (11) are function of m_{t-1}^{S} and m_{t-1}^{B} these two variables can completely be oscillatory according to (14) and (13).

Therefore, interest rate similar to the amount of deposits (savings) and loans in both loans and deposits markets can fluctuate.

Transfer of fluctuations from monetary sector to real sector

Although the transfer of monetary fluctuations to real sector can be clearly seen and understood, but in order to clarify the subject, we consider the equilibrium at macro level and its relation with interest rate fluctuations induced by the banking behavior. According to national accounting relationship we can write:

$$\begin{aligned} gdp &= con + inv + gov + ex - im \\ gde &= con + sav + tax + tr \\ gdp &= gde \end{aligned} \quad (16)$$

In which:

gdp = gde	Gross domestic product = Gross domestic expenditure
con	Consumption
inv	Investmnt
gov	Government expenditures
ex	Exports
im	Imports
sav	Saving
tax	Tax
tr	Transfer payments to outside

By solving equation (16), the equilibrium condition in macro economy will be obtained:

$$(inv - sav) + (gov - tax)(e + x - im - tr) = 0 \quad (17)$$

Foreign exchange markets relate international capital markets to real sector through foreign exchange rates and financial derivatives such as options, futures, forward contracts, and time deposit certificates where these markets also oscillatory affect the real sector through interest rate similarly, but for simplicity, we do not consider it and according to equations (15) and (11), we will only consider the two variables of investment and saving as functions of interest rates of saving deposits and loans (r^{S} and r^{L}). In other words, the equilibrium condition in the economy in time t will be as follows:

$$(\mathrm{inv_t}(\mathrm{r_t^L}) - \mathrm{sav_t}(\mathrm{r_t^S})) + (\mathrm{gov_t} - \mathrm{tax_t}) + (\mathrm{ex_t} - \mathrm{im_t} - \mathrm{tr_t}) = 0 \quad (18)$$

By replacing $\mathrm{r_t^S}$ and $\mathrm{r_t^L}$ from equations (15) and (11) in equilibrium condition, we will have:

$$(\mathrm{inv_t}\left(r^L(g(r^{S^{-1}}(r_{t-1}^L)))\right) - \mathrm{sav_t}\left(r^S(f(r^{L^{-1}}(r_{t-1}^L)))\right)) + (\mathrm{gov_t} - \mathrm{tax_t}) + (\mathrm{ex_t} - \mathrm{im_t} - \mathrm{tr_t}) = 0 \quad (19)$$

If the government has balanced fiscal policy which means $(\mathrm{gov_t} - \mathrm{tax_t}) = 0$ and the trade balance is also balanced ($(\mathrm{ex_t} - \mathrm{im_t} - \mathrm{tr_t}) = 0$), again the equilibrium (19) is a second order difference equation which can lead the economy into disequilibrium in different times. In other words, the mathematical behavior of equation (19) will be different regarding the behavioral characteristic of saving and loan contracts and the reaction of depositors and borrowers (investors) to interest rates of deposits and loans which can show any kind of oscillatory behavior. But this behavior cannot be expanding (diverging) or dampening (converging) forever, because disequilibrium tends to zero in the long run. Therefore, necessarily even if the equilibrium is a moving equilibrium, it should oscillate around its long run equilibrium and this phenomenon creates economic cycles.

It may be necessary to brief that by using fiscal policy $(\mathrm{gov_t} - \mathrm{tax_t})$ according to Keynes theory, we can compensate disequilibrium in equation (18). But by injecting financial resources to the economy equal to $(\mathrm{gov_t} - \mathrm{tax_t})>0$, the excess government budget in time t *Ceteris Paribus* will be government debt to the central bank at time t+1 and disrupts budget balance and the government has to levy more tax to compensate this deficit in the next period which leads to recession and disequilibrium and fluctuation in the next period. The same is true about the last parentheses of equation (19) about trade balance which is $(\mathrm{e + x - im - tr})$. By creating export incentives and import barriers for imports at time t, government increases the trade balance equal to $(\mathrm{ex_t} - \mathrm{im_t} - \mathrm{tr_t}) > 0$, and by increase of exchange rate in period t+1, exchange stability will be disrupted and we will face deficit in foreign trade which will have negative effects on the economy and creates fluctuations.

The capability of second order difference equation in explaining fluctuations

In order to clarify more second order difference equations' behavior, we will brief the time trend dynamism of linear non-homogenous difference equation with fixed coefficients. Consider a linear non-homogeneous second order difference equation with constant L and fixed coefficients of a and b:

$$y_{t+2} + ay_{t+1} + by_t = L \quad (20)$$

According to the amounts of a and b, the time trend of this equilibrium can be linear or non-linear for each Special Solution. By foregoing L for the time being, the Complementary Function will be as follow:

$$y_{t+2} + ay_{t+1} + by_t=0 \quad (21)$$

This difference equation has the following second order Characteristic Equation with roots x',x":

$$x^2 + ax + b = 0 \qquad x', x'' = (-a \pm \sqrt{a^2 - 4b})/2 \quad (22)$$

The Final Solution is in the form of the sum of "Special Solution" (y_p=M) (by solving the non-homogeneous equation) and "Complementary Function":

$$y_t = A(x')^t + B(x'')^t + y_p$$

If the equation has <u>two true positive or zero roots,</u> when both of these roots are true and less than one, by the increase of t, y_t tends to constant M (figure 2). But if x'=1 and x''<1, when t increases, y_t tends to A+M (figure 3). If x''=1 and x'<1, when t increases, y_t will finally tend to B+M. If one of the roots is bigger than the other (x'>1) and B and A are both positive, by increase of t, y_t will increase (figure 4).

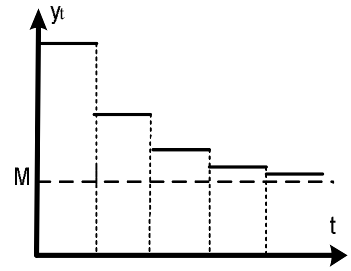


Figure 2

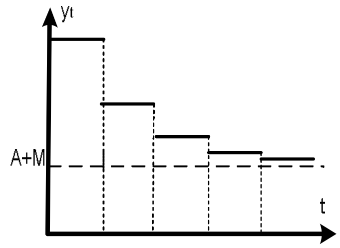


Figure 3

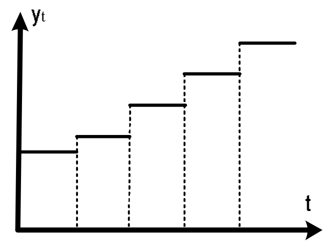


Figure 4

If A<0 and B >o, the result depends on whether x' or x'' is larger. At the preliminary stages, by increase of t, the amount of y_t is more affected by the term whose root is larger (figure 5).

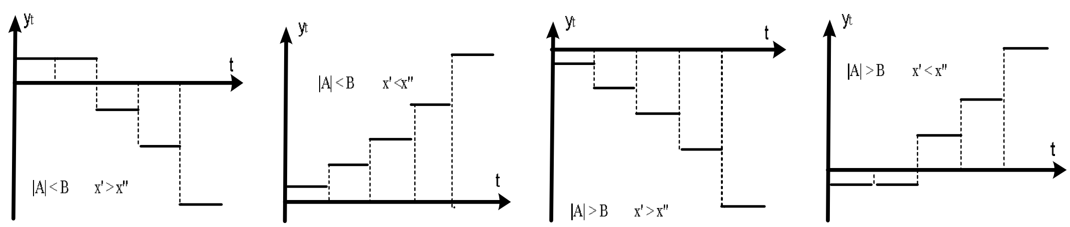


Figure 5

If <u>the two roots are true and negative (or zero)</u>, when x'', x'<0, when t is even, $(x'')^t$ and $(x')^t$ will be positive, and when t is odd, these items will be negative. This causes y_t to oscillate in successive periods. If the absolute value of both roots are less than one, 0<|x'|, |x''|<1; which means: -1<x', x''<0, by increase of t, $|A(x')^t|$, $|B(x'')^t$ will reduce and the amount of y_t tends towards M and finally will fit on line y_p= M (figure 6). If x'= -1 and -1<x''<0, $A(X')^t$ does not depend on the amount of t, but only on its being odd or even. When t is even, the last expression is equal to A and when it is odd, it will be equal to –A. Also by increase of t, the amount of $B(x'')^t$ will decrease and y_t oscillatory tends to $A(x')^t$+M and $A(-1)^t$+M (figure 7). When x',x''<-1 , y_t fluctuation will increase by the increase of t and the root with higher absolute value, will have a more effective role in fluctuations of y_t (figure 8).

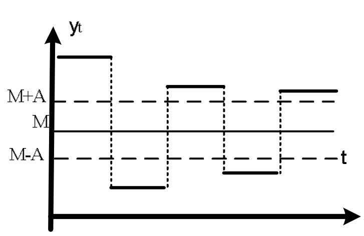


Figure 6

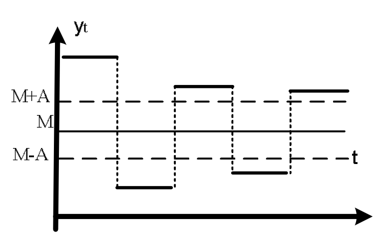


Figure 7

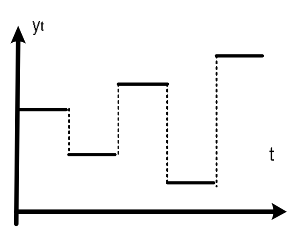


Figure 8

If <u>one root is positive and the other is negative</u>, the absolute value of the larger root will have more effects on y_t. The following cases may occur:

If 1>x'>0 and |x'|>| x", y_t will finally tend to M (figure 2).
If x'=1 and| |x'|>|x", y_t tends to A+M (figure 3).
If x'>1 and| x'|>|x"|, the increase of t will increase y_t (figure 4).
If -1<x'<0 and |x'|>|x"|, by increase of t, the amount of y_t will fluctuate and finally will tend to M (figure 6).
If x'=-1 and| x'|>|x"|, by the increase of t, y_t will oscillate around A+M and –A+M (figure 9).

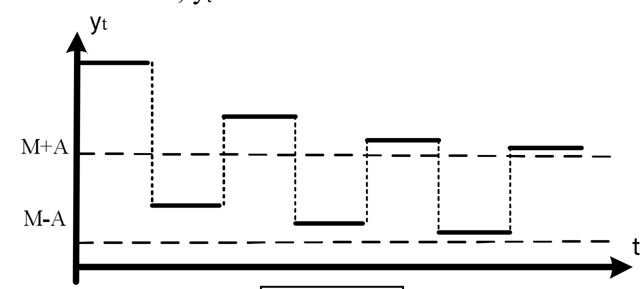


Figure 9

If x'<-1 and| x'|>|x", y_t will have expanding fluctuations in relation with M when t increases (figure 8).
If x'=-x", changes in y_t depends on the sign of A, B and larger absolute value of A in relation with B and vice versa.

If the root are double and real, the solution of the difference equation will be as follows:

$$y_t = Ax^t + Btx^t + M \qquad (23)$$

If 1>x>0, Btx^t will increase by the increase of t and will have an expanding time trend.
If x=1, and Btx^t = Bt, we will have a similar case to previous one.
If 0<x<1, Btx^t tends to zero when t increases. This trend is not compensated by the expanding trend of t and when t increases, x^t will be more than t and therefore, the term Btx^t will tend to zero.
If -1<x<0, we have a trend similar with the case 0<x<1, except that the term Btx^t has a fluctuation trend which tends to zero.
If x=-1, the trend of Btx^t is similar with x=1, but with fluctuations.
If x<-1, the result is similar with x>1, but with fluctuations.

If the roots are imaginary or complex, the solution of the difference equation will be:

$$y_t = D^t[E \text{ Cos } (tR)+F \text{ Sin } (tR)] + M \qquad (24)$$

where the roots of the complex characteristic equation will be c±di and its module will be $D=\sqrt{c^2+d^2}$. The term in the bracket has an iterated fluctuation in each 360°. In other words, y_t has a cycle equal to t=360°/R. Since Sin R=d/D, if for example R=60°, the range of the cyclical movement will be t=360°/60°.

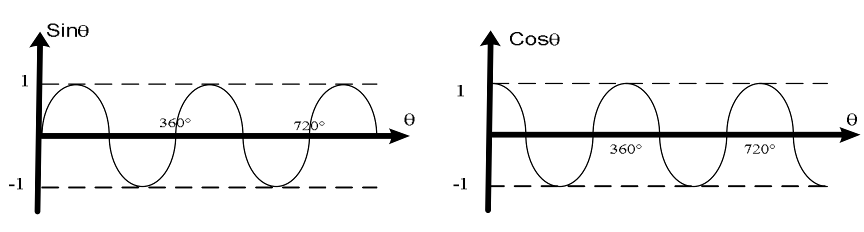


Figure 10

For, $\theta = tR$ the curves $Sin\theta$ and in $Cos\theta$ figure 10 have symmetrical cycles. Since amount of t is discrete, t may not be located at the beginning or middle of those cycles; therefore, figures 11, 12 and 13 look unbalanced. The term in the brackets of equation (24) has smooth fluctuations, but when reaches to power t, if D>1, by increase of t, D^t will increase and multiplied by the amount of the brackets in equation (24) will increase the fluctuations (figure 11). If D=1, D^t will also be equal to 1 and will have no effects on the amount of the bracket and y_t will have a monotonous (smooth) fluctuations (figure 12).

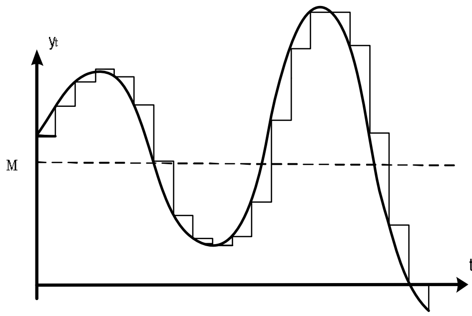


Figure 11

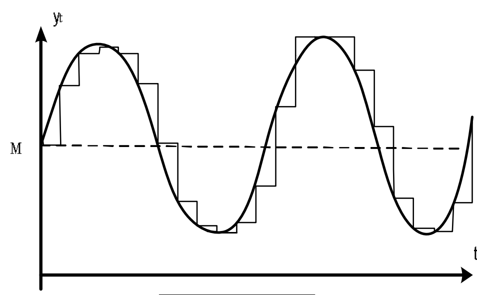


Figure 12

If D<1, when t increases, D^t decreases and fluctuations of y_t will tend to zero (figure 13). If we have a Special Solution, (this solution may be in the form of M, Mt or Mt^2), we define y_p as the temporary equilibrium of y_t. But if the special solution is not fixed, the equilibrium will be moving. For example, the Special Solution can be in the form of y_p=bt where the difference equation has no temporary equilibrium and bt is a moving equilibrium (figure 14).

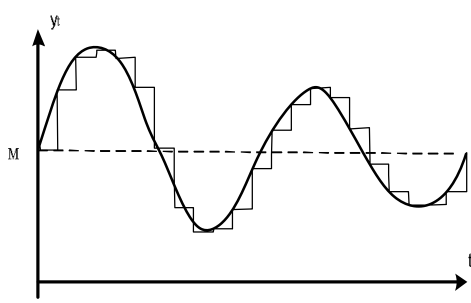


Figure 13

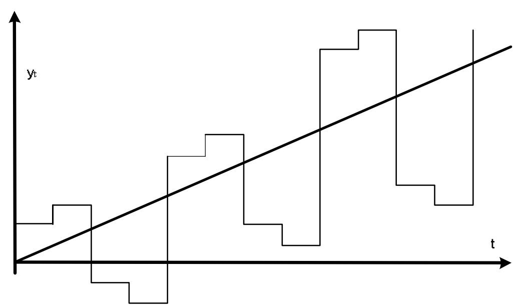


Figure 14

In general, for a second order homogenous difference equation with constant coefficients such as:

$$y_t + by_{t-1} + cy_{t-2} = 0 \quad (25)$$

With the help of figure 15 we can describe the time trend of y_t by considering the coefficients of difference equation (b, c) and the roots of characteristic equation.

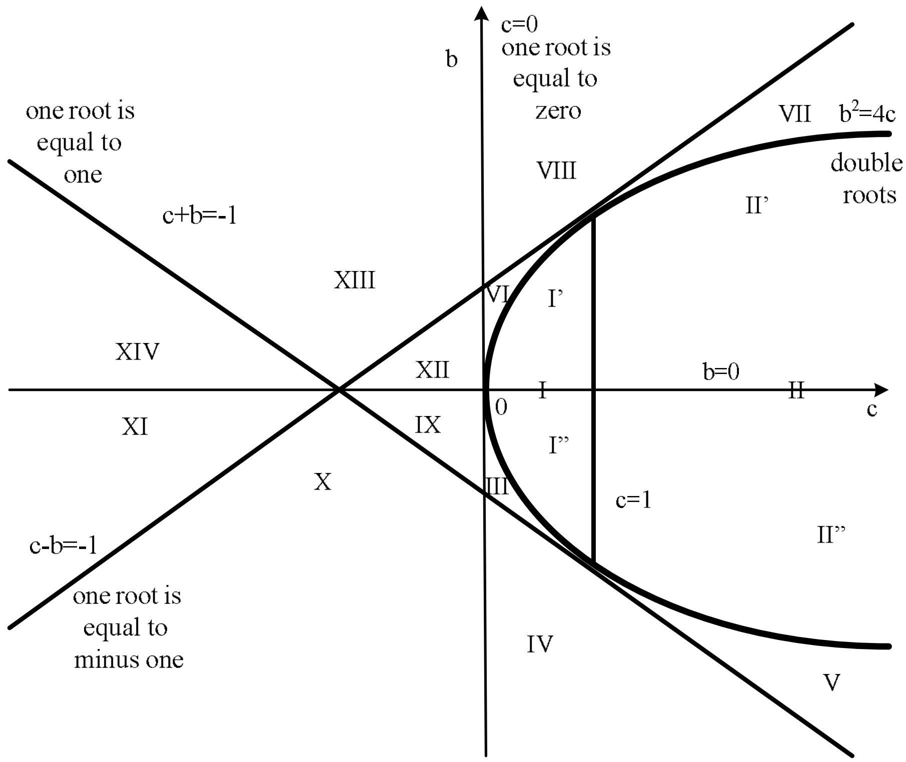


Figure 15

In this figure:

Area I: includes (I", I'): oscillatory dampening fluctuation (complex roots, module<1)
Area II: includes (II", II'): explosive oscillatory fluctuation (complex roots, module>1)
Area III: dampening and non-oscillatory (real roots, less than one)
Area IV: finally dampening and without oscillation (real roots, both positive, one larger than and the other less than one)
Area V: non-dampening and without oscillation (real roots and larger than one)
Area VI: oscillatory dampening (real roots, both negative with absolute values less than one)
Area VII: explosive with oscillation (real roots, both negative with absolute values larger than one)
Area VIII: finally explosive with oscillation (real roots, both positive and less than one)
Area IX: dampening without oscillation (real roots, both positive and less than one)
Area X: finally explosive without oscillation (real roots with opposite signs, the positive root is larger than one and the absolute value of the negative root is less than one)
Area XI: finally explosive without oscillation (real roots, with opposite signs, the absolute value of both roots are larger than one and the absolute value of the positive root is larger)
Area XII: dampening with oscillation (real roots with opposite signs, the absolute value of both roots less than one)
Area XIII: finally explosive with oscillation (real roots with opposite signs, the absolute value of the negative root is larger than one and the positive root is less than one)
Area XIV: finally explosive with oscillation (real roots with opposite signs, the absolute value of both roots are larger than one and the absolute value of the negative root is larger than the positive root)

In analyzing the time trend of y_t in a second order difference equation, the characteristics of being real roots, complex roots and double roots, their signs, the absolute value of them and being larger or smaller than one, are critical. In figure 15, many other areas can be enumerated and studied like all the points located on drawn lines and curve or at the intersection of them and so on, but since it is time consuming, we stop here[15].

All the above analyses are only about second order non-homogeneous difference equations with a constant term and fixed coefficients. But if the constant term is a function of time or the difference equation is not linear, or has a higher order, or the coefficients are functions of time, y_t will have different trends which create moving equilibriums with various fluctuations. The mathematical behavior of this kind of equations can be coinciding with difference equation in the previous discussion about interest rates and deposit and loan sources which create any kind of fluctuations in money and capital markets that can easily fluctuate the supply and demand in real sector through investment and consumption demands functions.

Empirical Investigations

To test the time structure of the interest rate the following interest rates were selected for a long period since 1948 till present[16]:

Discount Rate (End of Period):
Rate at which the Federal Reserve Bank of New York discounts eligible paper and makes advances to member banks. Borrowing from a Federal Reserve Bank may take the form either of discounts of short term commercial, industrial, and other financial paper or of advances against government securities and other eligible collateral. Federal Reserve advances to or discounts for member banks are usually of short maturity up to fifteen days.

Federal Funds Rate:
Weighted average rate at which banks borrow funds through New York brokers.

Commercial Paper Rate:
Rate on three-month commercial paper of nonfinancial firms. Rates are quoted on a discount basis and interpolated from data on certain commercial paper trades settled by the Depository Trust Company.

Treasury Bill Rate:
Weighted average yield on multiple-price auctions of 13-week treasury bills.

Certificate of Deposit Rate:
Average of dealer offering rates on nationally traded certificates of deposits.

Lending Rate:
Base rate charged by banks on short-term business loans.

Mortgage Rate:
Contract rate on 30-year fixed-rate first mortgages.

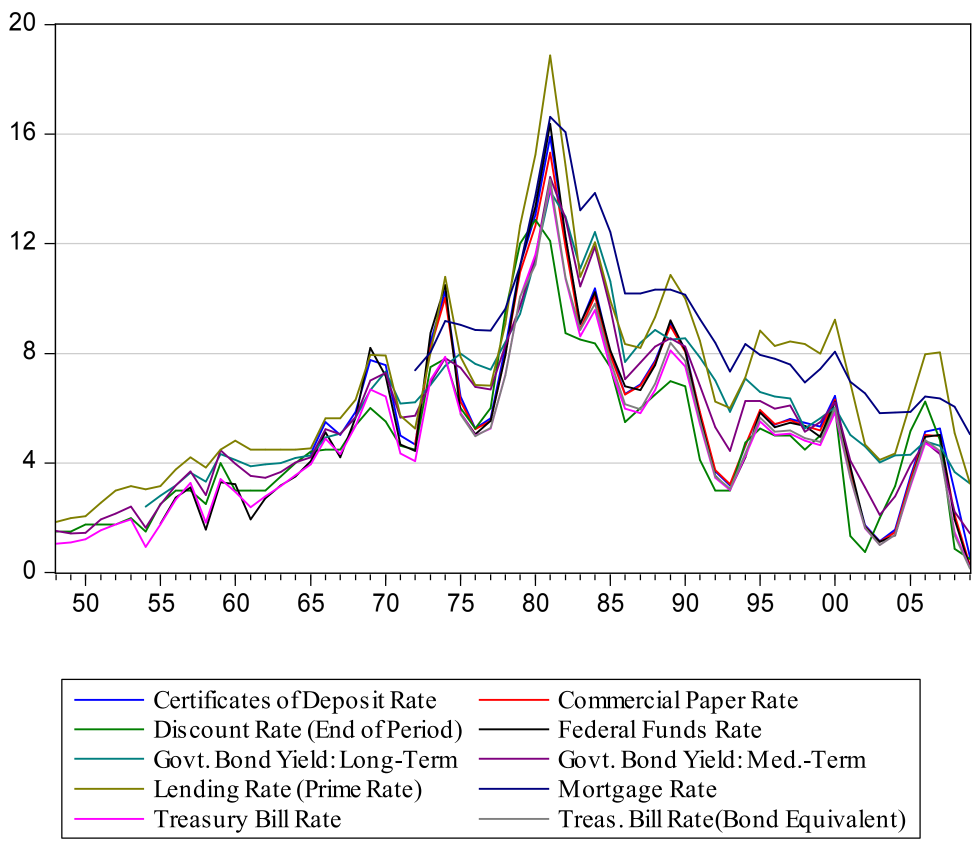
Government Bond Yield: Long-Term and Short-Term:

[15] - For more information about time behavior of difference equations look through "Difference Equations and the Stability of Equilibrium Dynamism", Bijan Bidabad. http://www.bidabad.com/doc/difference-equations.pdf
[16] - International Financial Statistics, Country Note 2010, USA, www.imfstatistics.org

Yield on actively traded treasury issues adjusted to constant maturities. Yield on treasury securities at constant maturity are interpolated by the U.S. Treasury from the daily yield curve. This curve, which relates the yield on a security to its time to maturity, is based on the closing market bid yields on actively traded treasury securities in the over-the-counter market. These market yields are calculated from composites of quotations obtained by the Federal Reserve Bank of New York. Medium-Term rate refers to three-year constant maturities. Long-term rate refers to ten year constant maturities.

Figure 16. shows the time path of different interest rates during the sample period.

Figure 16. Different interest rates time trend. USA (%) for 1948-2009



To show the oscillatory natures of the interest rates we need to test that the equations (11) and (15) are true. That is these two equations are difference equations with (at least) orders 2 or more. Using data of the Unites States of America we try to show that equation (11) and (15) are oscillatory and therefore the source of oscillation will be transferred to equations (18) and (19) which are the equilibrium condition of the macro-economy. We can conclude that the source of oscillation is emanated from interest rates to real sector, and the interest rates are the source of cycles in the economy.

Though the saving and loan interest rates are sufficed to be tested we test that various interest rates obey oscillatory behaviors. Ten types of short term and medium term and long term interest rates were selected. We fitted a second order linear non-homogenous difference equation to all 10 interest rates as given by (20).

Table 1. Estimation results for ten estimated equations

Dependent Variable	$Y_t = ß_0 + ß_1 . Yt_{-1} + ß_2 . Y_{t-2}$						
	Sample	**obs.**	**Y_t**	**$ß_0$**	**$ß_1$**	**$ß_2$**	**R^2**
Certificates of Deposit Rate (secondary market-3 month)	1967-2009	43	CDR	1.412	1.173	-0.405	0.714
			t-Stat.	2.124	7.828	-2.675	
Commercial Paper Rate	1974-2009	36	CPR	1.092	1.208	-0.406	0.761
			t-Stat.	1.609	7.807	-2.564	
Discount Rate (End of Period)	1950-2009	60	DR	0.877	1.168	-0.349	0.774
			t-Stat.	2.387	9.376	-2.786	
Federal Funds Rate	1957-2009	53	FFR	1.159	1.121	-0.332	0.721
			t-Stat.	2.241	8.255	-2.445	
Lending Rate (Prime Rate)	1950-2009	60	LPR	1.193	1.195	-0.364	0.799
			t-Stat.	2.553	9.559	-2.962	
Treasury Bill Rate(Bond Equivalent-3 month)	1977-2009	33	TBRBE	0.920	1.212	-0.384	0.768
			t-Stat.	1.412	7.048	-2.153	
Mortgage Rate	1974-2009	36	MR	0.713	1.301	-0.386	0.874
			t-Stat.	1.140	7.983	-2.339	
Treasury Bill Rate	1950-2009	60	TBR	0.738	1.173	-0.330	0.792
			t-Stat.	2.126	9.257	-2.614	
Govt. Bond Yield: Long Term (10 year)	1956-2009	54	GBYLT	0.511	1.103	-0.180	0.880
			t-Stat.	1.491	7.973	-1.319	
Govt. Bond Yield: Medium Term (3 year)	1950-2009	60	GBYMT	0.539	1.127	-0.222	0.856
			t-Stat.	1.656	8.668	-1.718	

As table 1 shows all estimated parameters are statistically significant and prove that a second order linear difference equation time structure exist for all 10 selected interest rates.

Table 2. Characteristic roots of the second order linear difference equations

Variables		$y_{t+2} = a.y_{t+1} + b.y_t + L$				real	imag.		Characteristic roots
		a	**b**	**L**	**roots**	**c**	**d**	**D**	**x', x"**
CDR	Certificates of Deposit Rate (secondary market-3 month)	-1.173	0.405	1.412	complex	0.586	0.865	1.045	0.586±0.865i
CPR	Commercial Paper Rate	-1.208	0.406	1.092	complex	0.604	0.878	1.065	0.603±0.877i
DR	Discount Rate (End of Period)	-1.168	0.349	0.877	complex	0.584	0.831	1.016	0.584±0.830i
FFR	Federal Funds Rate	-1.121	0.332	1.159	complex	0.561	0.804	0.980	0.560±0.803i
LPR	Lending Rate (Prime Rate)	-1.195	0.364	1.193	complex	0.598	0.849	1.039	0.597±0.849i
TBRBE	Treasury Bill Rate(Bond Equivalent-3 month)	-1.212	0.384	0.920	complex	0.606	0.867	1.058	0.606±0.866i
MR	Mortgage Rate	-1.301	0.386	0.713	real				0.843, 0.458
TBR	Treasury Bill Rate	-1.173	0.330	0.738	real				0.705, 0.469
GBYLT	Govt. Bond Yield: Long Term (10 year)	-1.103	0.180	0.511	real				0.903, 0.200
GBYMT	Govt. Bond Yield: Medium Term (3 year)	-1.127	0.222	0.539	real				0.872, 0.255

As table 2 shows all short term interest rates' second order homogeneous linear difference equations have complex characteristic roots; but the roots for medium and long terms interest rates are real. This proves that the source of fluctuations in real economy comes from short term interest rates, though medium and long terms interest rates have real characteristic roots and dampening behaviors.

Since value of D in short term rates is less than and nearly equal and greater than one the forms of oscillations will be as shown by figures 11 to 13. On the other side, the medium and long term interest rates have real characteristic roots which each one of the pairs is near to one and another is much less than one. Therefore the medium and long term interest rates have time path similar to figures 2 and 3 and have dampening behavior.

Details of Estimations

Dependent Variable: CDR
Method: Least Squares
Date: 12/22/10 Time: 17:31
Sample (adjusted): 1967 2009
Included observations: 43 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.412185	0.664762	2.124347	0.0399
CDR(-1)	1.172582	0.149786	7.828360	0.0000
CDR(-2)	-0.404723	0.151324	-2.674555	0.0108

R-squared	0.714088	Mean dependent var	6.406395
Adjusted R-squared	0.699793	S.D. dependent var	3.189471
S.E. of regression	1.747549	Akaike info criterion	4.021520
Sum squared resid	122.1571	Schwarz criterion	4.144394
Log likelihood	-83.46267	Hannan-Quinn criter.	4.066832
F-statistic	49.95163	Durbin-Watson stat	1.681986
Prob(F-statistic)	0.000000		

Dependent Variable: CPR
Method: Least Squares
Date: 12/22/10 Time: 17:32
Sample (adjusted): 1974 2009
Included observations: 36 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.091869	0.678635	1.608920	0.1172
CPR(-1)	1.207907	0.154712	7.807449	0.0000
CPR(-2)	-0.405724	0.158239	-2.564003	0.0151

R-squared	0.760982	Mean dependent var	6.269028
Adjusted R-squared	0.746496	S.D. dependent var	3.364833
S.E. of regression	1.694166	Akaike info criterion	3.971914
Sum squared resid	94.71658	Schwarz criterion	4.103874
Log likelihood	-68.49445	Hannan-Quinn criter.	4.017971
F-statistic	52.53240	Durbin-Watson stat	1.808214
Prob(F-statistic)	0.000000		

Dependent Variable: DR
Method: Least Squares
Date: 12/22/10 Time: 17:33
Sample (adjusted): 1950 2009
Included observations: 60 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.876916	0.367370	2.387009	0.0203
DR(-1)	1.168123	0.124589	9.375823	0.0000
DR(-2)	-0.349087	0.125290	-2.786234	0.0072

R-squared	0.773563	Mean dependent var	4.900667
Adjusted R-squared	0.765618	S.D. dependent var	2.721932
S.E. of regression	1.317770	Akaike info criterion	3.438465
Sum squared resid	98.98145	Schwarz criterion	3.543182
Log likelihood	-100.1539	Hannan-Quinn criter.	3.479426
F-statistic	97.36282	Durbin-Watson stat	1.821936
Prob(F-statistic)	0.000000		

Dependent Variable: FFR
Method: Least Squares
Date: 12/22/10 Time: 17:33
Sample (adjusted): 1957 2009
Included observations: 53 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.159006	0.517073	2.241475	0.0295
FFR(-1)	1.121160	0.135817	8.254928	0.0000
FFR(-2)	-0.331783	0.135677	-2.445394	0.0180

R-squared	0.721248	Mean dependent var	5.688585
Adjusted R-squared	0.710098	S.D. dependent var	3.275746
S.E. of regression	1.763744	Akaike info criterion	4.027693
Sum squared resid	155.5396	Schwarz criterion	4.139219
Log likelihood	-103.7339	Hannan-Quinn criter.	4.070581
F-statistic	64.68561	Durbin-Watson stat	1.766529
Prob(F-statistic)	0.000000		

Dependent Variable: GBYLT
Method: Least Squares
Date: 12/22/10 Time: 17:34
Sample (adjusted): 1956 2009
Included observations: 54 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.510742	0.342630	1.490651	0.1422
GBYLT(-1)	1.102660	0.138297	7.973138	0.0000
GBYLT(-2)	-0.180204	0.136643	-1.318792	0.1931

R-squared	0.879766	Mean dependent var	6.543981
Adjusted R-squared	0.875050	S.D. dependent var	2.604271
S.E. of regression	0.920563	Akaike info criterion	2.726290

Sum squared resid	43.21926	Schwarz criterion	2.836789
Log likelihood	-70.60984	Hannan-Quinn criter.	2.768905
F-statistic	186.5856	Durbin-Watson stat	1.934043
Prob(F-statistic)	0.000000		

Dependent Variable: GBYMT
Method: Least Squares
Date: 12/22/10 Time: 17:35
Sample (adjusted): 1950 2009
Included observations: 60 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.539291	0.325607	1.656265	0.1032
GBYMT(-1)	1.127038	0.130020	8.668214	0.0000
GBYMT(-2)	-0.222132	0.129317	-1.717740	0.0913

R-squared	0.855818	Mean dependent var	5.699500
Adjusted R-squared	0.850759	S.D. dependent var	2.914363
S.E. of regression	1.125868	Akaike info criterion	3.123693
Sum squared resid	72.25200	Schwarz criterion	3.228410
Log likelihood	-90.71078	Hannan-Quinn criter.	3.164653
F-statistic	169.1670	Durbin-Watson stat	1.902443
Prob(F-statistic)	0.000000		

Dependent Variable: LPR
Method: Least Squares
Date: 12/22/10 Time: 17:35
Sample (adjusted): 1950 2009
Included observations: 60 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.193421	0.467540	2.552554	0.0134
LPR(-1)	1.195410	0.125060	9.558684	0.0000
LPR(-2)	-0.364144	0.122934	-2.962120	0.0044

R-squared	0.798558	Mean dependent var	7.086450
Adjusted R-squared	0.791490	S.D. dependent var	3.319712
S.E. of regression	1.515878	Akaike info criterion	3.718574
Sum squared resid	130.9795	Schwarz criterion	3.823291
Log likelihood	-108.5572	Hannan-Quinn criter.	3.759534
F-statistic	112.9797	Durbin-Watson stat	1.777337
Prob(F-statistic)	0.000000		

Dependent Variable: MR
Method: Least Squares
Date: 12/22/10 Time: 17:36
Sample (adjusted): 1974 2009
Included observations: 36 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.713471	0.625743	1.140198	0.2624
MR(-1)	1.300689	0.162941	7.982556	0.0000

MR(-2)	-0.385800	0.164963	-2.338703	0.0256

R-squared	0.874263	Mean dependent var	9.109667
Adjusted R-squared	0.866642	S.D. dependent var	2.883206
S.E. of regression	1.052895	Akaike info criterion	3.020619
Sum squared resid	36.58339	Schwarz criterion	3.152579
Log likelihood	-51.37115	Hannan-Quinn criter.	3.066677
F-statistic	114.7258	Durbin-Watson stat	1.893167
Prob(F-statistic)	0.000000		

Dependent Variable: TBR
Method: Least Squares
Date: 12/22/10 Time: 17:36
Sample (adjusted): 1950 2009
Included observations: 60 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.738135	0.347145	2.126301	0.0378
TBR(-1)	1.173308	0.126742	9.257450	0.0000
TBR(-2)	-0.330272	0.126339	-2.614173	0.0114

R-squared	0.791526	Mean dependent var	4.801517
Adjusted R-squared	0.784212	S.D. dependent var	2.837310
S.E. of regression	1.318017	Akaike info criterion	3.438840
Sum squared resid	99.01856	Schwarz criterion	3.543557
Log likelihood	-100.1652	Hannan-Quinn criter.	3.479800
F-statistic	108.2080	Durbin-Watson stat	1.804008
Prob(F-statistic)	0.000000		

Dependent Variable: TBRBE
Method: Least Squares
Date: 12/22/10 Time: 17:37
Sample (adjusted): 1977 2009
Included observations: 33 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.920158	0.651453	1.412471	0.1681
TBRBE(-1)	1.212030	0.171978	7.047595	0.0000
TBRBE(-2)	-0.383979	0.178377	-2.152630	0.0395

R-squared	0.768303	Mean dependent var	5.760152
Adjusted R-squared	0.752857	S.D. dependent var	3.223851
S.E. of regression	1.602689	Akaike info criterion	3.867750
Sum squared resid	77.05833	Schwarz criterion	4.003797
Log likelihood	-60.81788	Hannan-Quinn criter.	3.913526
F-statistic	49.73984	Durbin-Watson stat	1.861321
Prob(F-statistic)	0.000000		

Data

obs	CDR	CPR	DR	FFR	GBYLT	GBYMT	LPR	MR	TBR	TBRBE
1948	NA	NA	1.500000	NA	NA	1.520000	1.854000	NA	1.041000	NA
1949	NA	NA	1.500000	NA	NA	1.430000	2.000000	NA	1.101000	NA
1950	NA	NA	1.750000	NA	NA	1.450000	2.069000	NA	1.218000	NA
1951	NA	NA	1.750000	NA	NA	1.940000	2.555000	NA	1.553000	NA
1952	NA	NA	1.750000	NA	NA	2.140000	3.000000	NA	1.768000	NA
1953	NA	NA	2.000000	NA	NA	2.420000	3.169000	NA	1.940000	NA
1954	NA	NA	1.500000	NA	2.402000	1.632000	3.053000	NA	0.950000	NA
1955	NA	NA	2.500000	1.785000	2.817000	2.471000	3.157000	NA	1.747000	NA
1956	NA	NA	3.000000	2.728000	3.183000	3.187000	3.770000	NA	2.659000	NA
1957	NA	NA	3.000000	3.105000	3.648000	3.686000	4.202000	NA	3.264000	NA
1958	NA	NA	2.500000	1.573000	3.316000	2.838000	3.833000	NA	1.838000	NA
1959	NA	NA	4.000000	3.305000	4.333000	4.460000	4.478000	NA	3.413000	NA
1960	NA	NA	3.000000	3.216000	4.117000	3.981000	4.821000	NA	2.947000	NA
1961	NA	NA	3.000000	1.955000	3.883000	3.535000	4.500000	NA	2.376000	NA
1962	NA	NA	3.000000	2.708000	3.946000	3.472000	4.500000	NA	2.778000	NA
1963	NA	NA	3.500000	3.178000	4.003000	3.669000	4.500000	NA	3.157000	NA
1964	NA	NA	4.000000	3.497000	4.187000	4.031000	4.500000	NA	3.553000	NA
1965	4.344000	NA	4.420000	4.073000	4.283000	4.218000	4.535000	NA	3.948000	NA
1966	5.483000	NA	4.500000	5.111000	4.923000	5.230000	5.625000	NA	4.882000	NA
1967	5.021000	NA	4.500000	4.220000	5.073000	5.039000	5.633000	NA	4.332000	NA
1968	5.859000	NA	5.360000	5.657000	5.646000	5.685000	6.313000	NA	5.347000	NA
1969	7.761000	NA	6.000000	8.204000	6.671000	7.017000	7.952000	NA	6.688000	NA
1970	7.564000	NA	5.520000	7.181000	7.348000	7.287000	7.910000	NA	6.437000	NA
1971	5.005000	NA	4.630000	4.661000	6.159000	5.654000	5.723000	NA	4.340000	NA
1972	4.666000	4.663000	4.500000	4.431000	6.210000	5.721000	5.248000	7.384000	4.069000	NA
1973	8.416000	8.203000	7.500000	8.728000	6.843000	6.955000	8.022000	8.043000	7.027000	NA
1974	10.24400	10.01400	7.810000	10.50300	7.557000	7.823000	10.79800	9.189000	7.875000	NA
1975	6.437000	6.250000	6.000000	5.824000	7.988000	7.490000	7.863000	9.041000	5.825000	5.776000
1976	5.268000	5.237000	5.250000	5.045000	7.611000	6.771000	6.840000	8.865000	4.999000	4.990000
1977	5.641000	5.545000	6.000000	5.538000	7.419000	6.686000	6.824000	8.841000	5.264000	5.291000
1978	8.222000	7.941000	9.500000	7.931000	8.410000	8.289000	9.057000	9.637000	7.223000	7.191000
1979	11.22500	10.96800	12.00000	11.19400	9.443000	9.715000	12.66600	11.19300	10.04300	10.07300
1980	13.06700	12.65800	12.87000	13.35600	11.46000	11.54800	15.26600	13.77000	11.61500	11.23600
1981	15.91100	15.32500	12.10000	16.37800	13.91100	14.43900	18.87000	16.63300	14.07800	14.35100
1982	12.27100	11.89300	8.730000	12.25800	13.00100	12.92200	14.86100	16.08400	10.72500	10.77000
1983	9.067000	8.878000	8.500000	9.087000	11.10500	10.44700	10.79400	13.22800	8.620000	8.867000
1984	10.36500	10.09700	8.370000	10.22500	12.43800	11.89300	12.04300	13.87000	9.573000	9.813000
1985	8.048000	7.954000	7.500000	8.101000	10.62300	9.643000	9.933000	12.42400	7.489000	7.725000
1986	6.518000	6.495000	5.500000	6.805000	7.683000	7.064000	8.333000	10.17800	5.973000	6.153000
1987	6.861000	6.813000	6.000000	6.658000	8.384000	7.668000	8.203000	10.19600	5.826000	5.954000
1988	7.728000	7.656000	6.500000	7.568000	8.846000	8.257000	9.315000	10.33800	6.672000	6.879000
1989	9.085000	8.989000	7.000000	9.217000	8.499000	8.558000	10.87300	10.32400	8.114000	8.391000
1990	8.148000	8.061000	6.790000	8.099000	8.550000	8.254000	10.00900	10.13000	7.510000	7.742000
1991	5.835000	5.865000	4.110000	5.688000	7.858000	6.813000	8.463000	9.249000	5.409000	5.534000
1992	3.682000	3.753000	3.000000	3.522000	7.010000	5.307000	6.252000	8.401000	3.460000	3.508000
1993	3.174000	3.224000	3.000000	3.023000	5.873000	4.443000	6.000000	7.330000	3.019000	3.064000
1994	4.629000	4.659000	4.750000	4.202000	7.080000	6.264000	7.138000	8.356000	4.270000	4.348000
1995	5.917000	5.933000	5.250000	5.837000	6.580000	6.263000	8.829000	7.955000	5.513000	5.653000
1996	5.390000	5.413000	5.000000	5.298000	6.438000	5.990000	8.271000	7.806000	5.024000	5.137000
1997	5.616000	5.571000	5.000000	5.460000	6.353000	6.102000	8.442000	7.596000	5.070000	5.198000
1998	5.467000	5.341000	4.500000	5.353000	5.264000	5.138000	8.354000	6.944000	4.819000	4.901000
1999	5.330000	5.183000	5.000000	4.970000	5.637000	5.485000	7.994000	7.429000	4.658000	4.768000
2000	6.456000	6.305000	6.000000	6.236000	6.029000	6.217000	9.233000	8.063000	5.839000	5.998000

2001	3.687000	3.608000	1.330000	3.888000	5.018000	4.081000	6.922000	6.973000	3.452000	3.477000
2002	1.726000	1.687000	0.750000	1.667000	4.611000	3.100000	4.675000	6.538000	1.613000	1.633000
2003	1.151000	1.113000	2.000000	1.128000	4.015000	2.107000	4.123000	5.819000	1.013000	1.021000
2004	1.563000	1.485000	3.150000	1.351000	4.274000	2.780000	4.340000	5.843000	1.373000	1.389000
2005	3.512000	3.379000	5.160000	3.212000	4.290000	3.925000	6.189000	5.866000	3.152000	3.213000
2006	5.153000	5.026000	6.250000	4.963000	4.792000	4.769000	7.958000	6.411000	4.722000	4.847000
2007	5.268000	4.987000	4.830000	5.019000	4.629000	4.338000	8.050000	6.343000	4.410000	4.452000
2008	2.965000	2.117000	0.860000	1.928000	3.667000	2.236000	5.088000	6.042000	1.460000	1.365000
2009	0.556000	0.262000	0.500000	0.160000	3.257000	1.427000	3.250000	5.043000	0.160000	0.143000

Source: International Financial Statistics, www.imfstatistics.org

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